Higher-Order Functions

Announcements

Designing Functions

Describing Functions

```
def square(x):
    """Return X * X."""
```

A function's *domain* is the set of all inputs it might possibly take as arguments.

x is a number

A function's *range* is the set of output values it might possibly return.

square returns a non-negative real number

A pure function's *behavior* is the relationship it creates between input and output.

square returns the square of x

A Guide to Designing Function

Give each function exactly one job, but make it apply to many related situations

```
>>> round(1.23)     >>> round(1.23, 1)     >>> round(1.23, 0)     >>> round(1.23, 5)
1                   1.2                    1                      1.23
```

Don’t repeat yourself (DRY). Implement a process just once, but execute it many times.

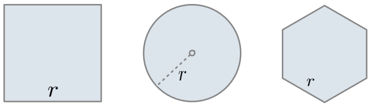
(Demo)

Generalization

Generalizing Patterns with Arguments

Regular geometric shapes relate length and area.

Shape:



Area: $\boxed{1} \cdot r^2$ $\boxed{\pi} \cdot r^2$ $\boxed{\frac{3\sqrt{3}}{2}} \cdot r^2$

Finding common structure allows for shared implementation

(Demo)

Higher-Order Functions

Generalizing Over Computational Processes

The common structure among functions may be a computational process, rather than a number.

$$\sum_{k=1}^{5} \boxed{k} = 1+2+3+4+5 \qquad = 15$$

$$\sum_{k=1}^{5} \boxed{k^3} = 1^3+2^3+3^3+4^3+5^3 \qquad = 225$$

$$\sum_{k=1}^{5} \boxed{\frac{8}{(4k-3)\cdot(4k-1)}} = \frac{8}{3}+\frac{8}{35}+\frac{8}{99}+\frac{8}{195}+\frac{8}{323} \qquad = 3.04$$

(Demo)

Summation Example

```
def cube(k):
    return pow(k, 3)

def summation(n, term):
    """Sum the first n terms of a sequence.

    >>> summation(5, cube)
    225
    """
    total, k = 0, 1
    while k <= n:
        total, k = total + term(k), k + 1
    return total
```

Function of a single argument (*not called "term"*)

A formal parameter that will be bound to a function

The cube function is passed as an argument value

0 + 1 + 8 + 27 + 64 + 125

The function bound to term gets called here

Functions as Return Values

(Demo)

Locally Defined Functions

Functions defined within other function bodies are bound to names in a local frame

A function that returns a function

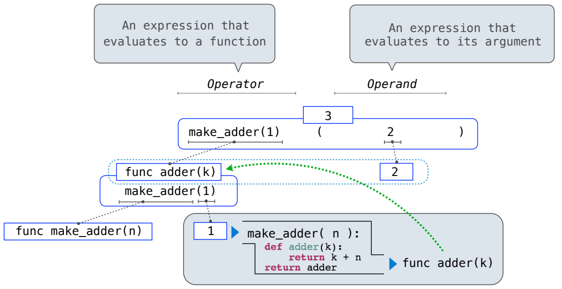
```
def make_adder(n):
    """Return a function that takes one argument k and returns k + n.

    >>> add_three = make_adder(3)
    >>> add_three(4)
    7
    """
    def adder(k):
        return k + n
    return adder
```

The name add_three is bound to a function

A def statement within another def statement

Can refer to names in the enclosing function

Call Expressions as Operator Expressions



Lambda Expressions

(Demo)

Lambda Expressions

```
>>> x = 10

>>> square = x * x

>>> square = lambda x: x * x
```

An expression: this one evaluates to a number

Also an expression: evaluates to a function

Important: No "return" keyword!

A function

with formal parameter x

that returns the value of "x * x"

```
>>> square(4)
16
```

Must be a single expression

Lambda expressions are not common in Python, but important in general

Lambda expressions in Python cannot contain statements at all!

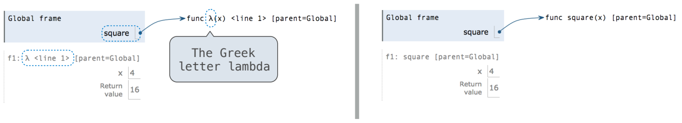
Lambda Expressions Versus Def Statements

```
square = lambda x: x * x
```

VS

```
def square(x):
    return x * x
```

- Both create a function with the same domain, range, and behavior.
- Both bind that function to the name square.
- Only the def statement gives the function an intrinsic name, which shows up in environment diagrams but doesn't affect execution (unless the function is printed).



Global frame
square
func λ(x) <line 1> [parent=Global]
f1: λ <line 1> [parent=Global]
x 4
Return value 16
The Greek letter lambda
Global frame
square
func square(x) [parent=Global]
f1: square [parent=Global]
x 4
Return value 16