Environments

Announcements

Environments for Higher-Order Functions

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Environment diagrams describe how higher-order functions work!

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Environment diagrams describe how higher-order functions work!

(Demo)

Names can be Bound to Functional Arguments

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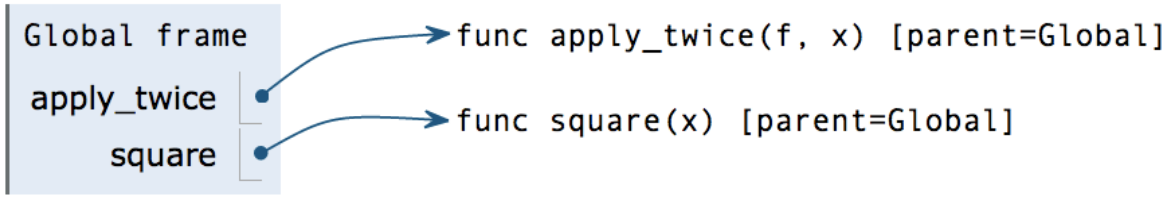

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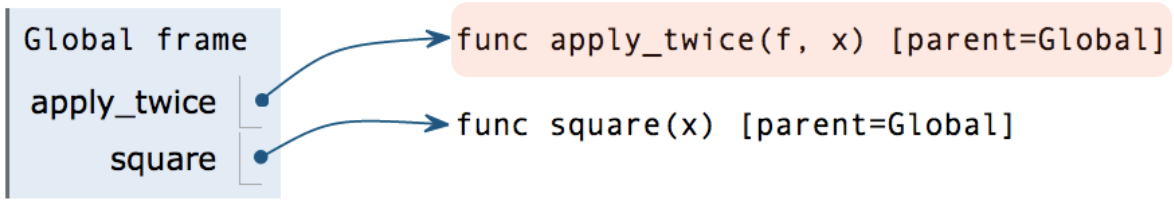

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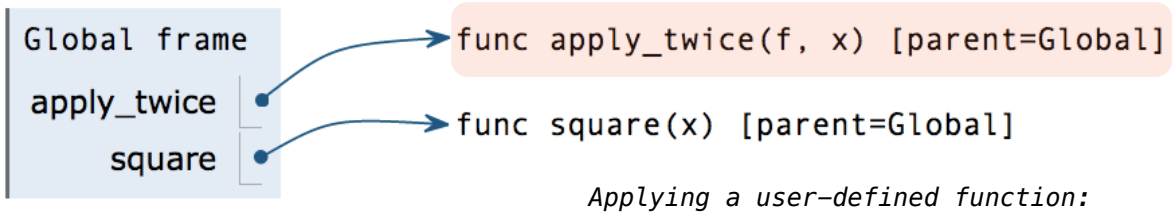

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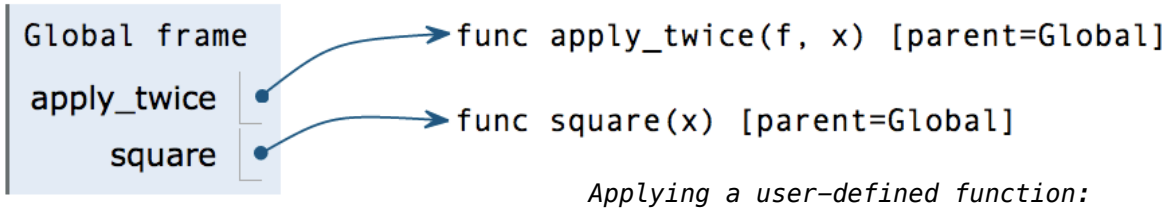

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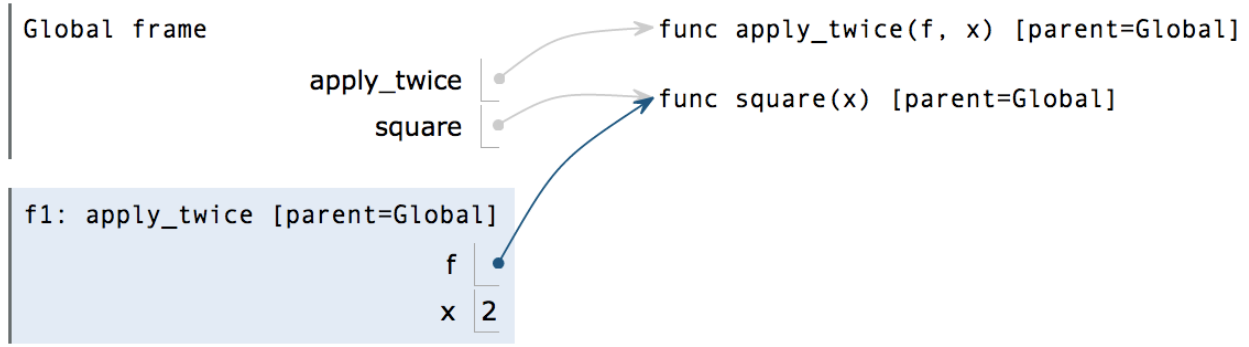

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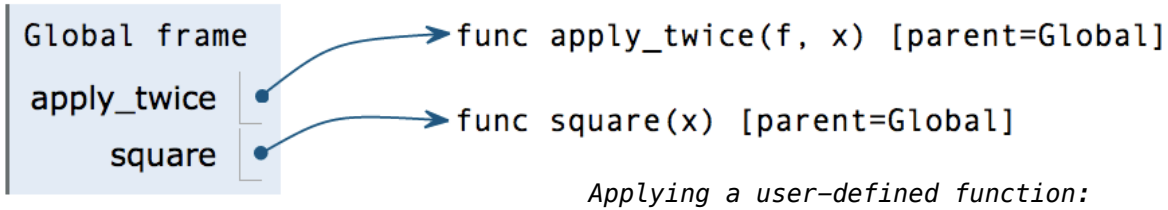

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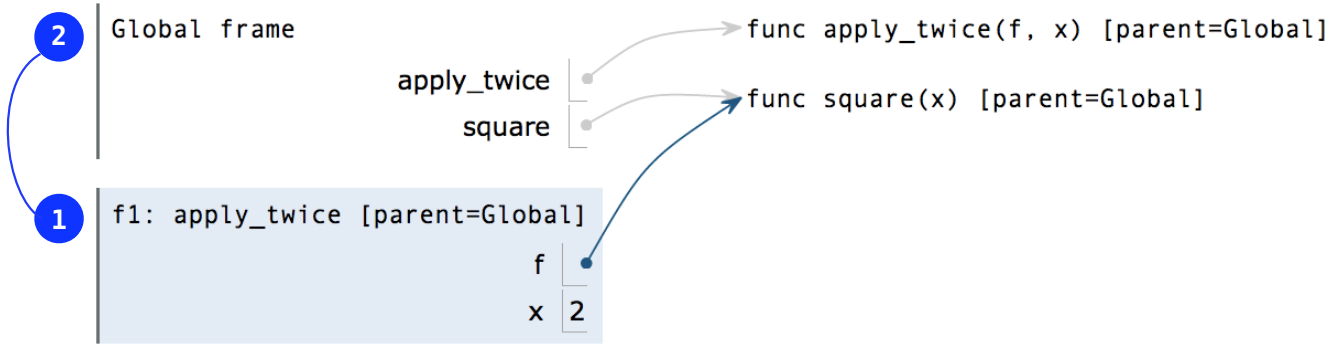

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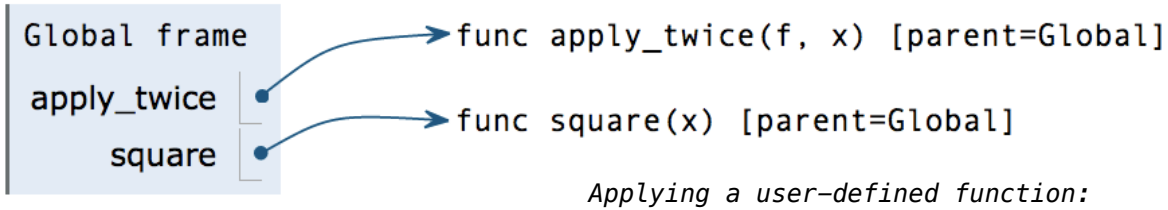

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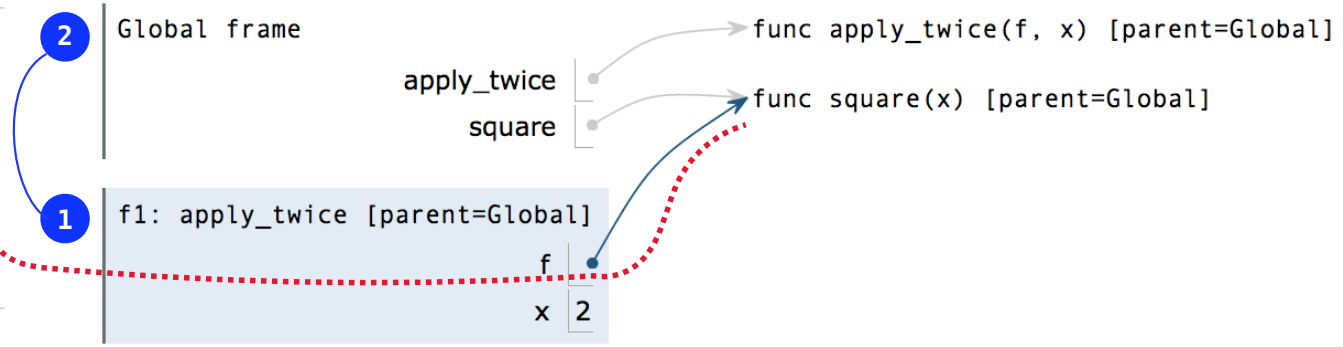

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Environments for Nested Definitions

(Demo)

Environment Diagrams for Nested Def Statements

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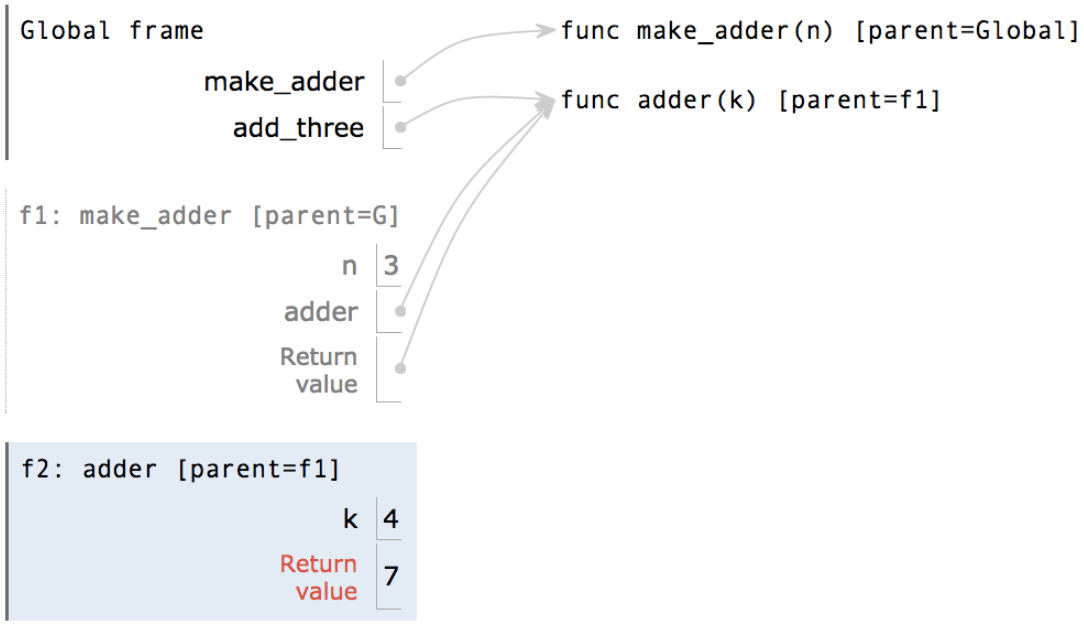

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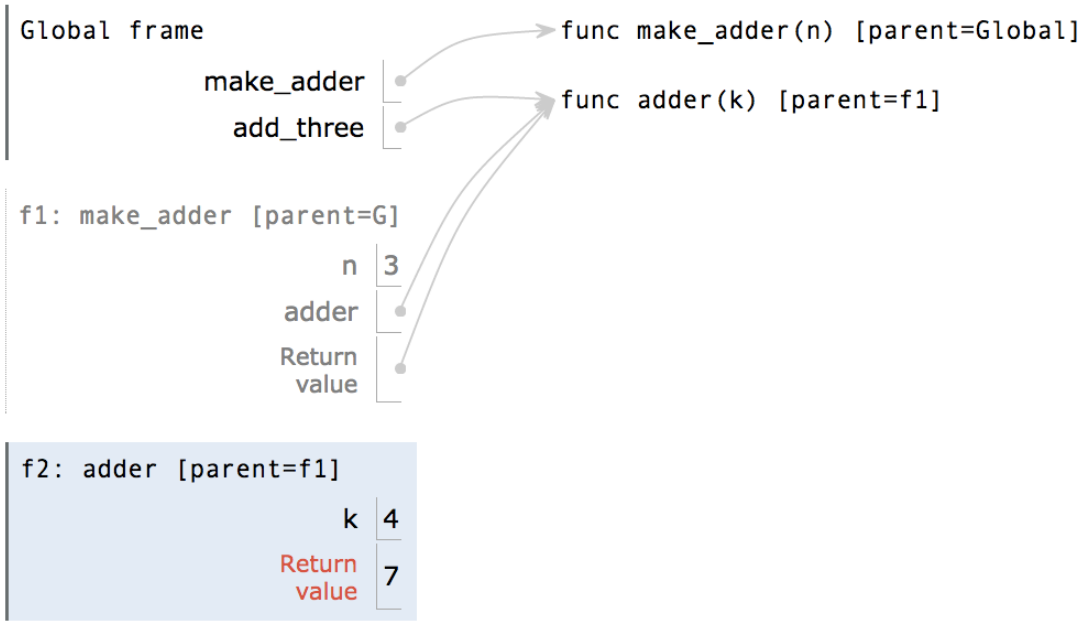

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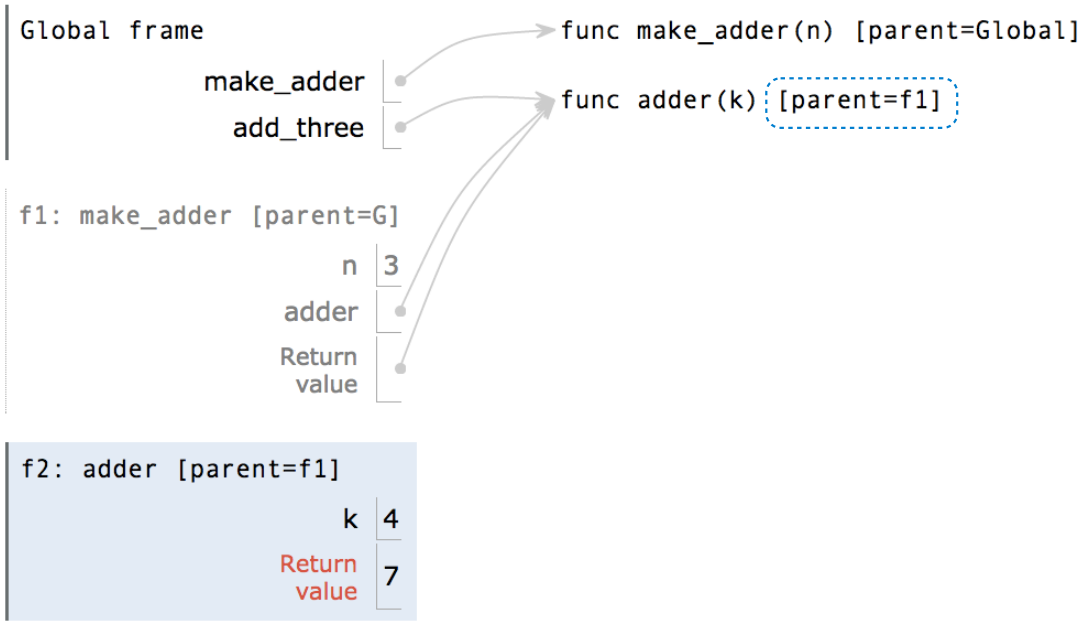

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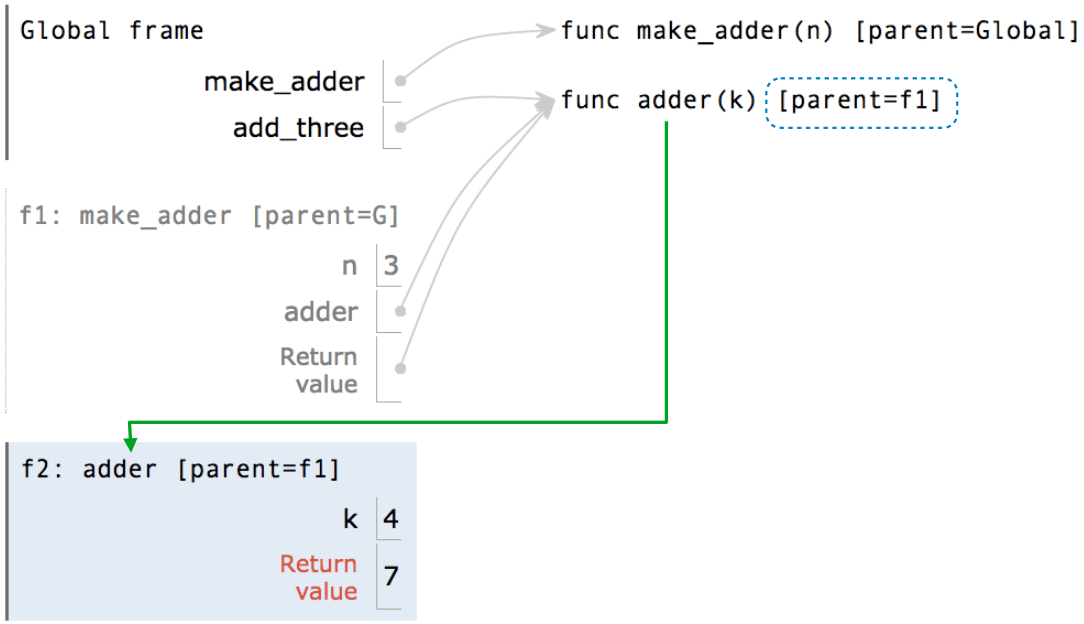

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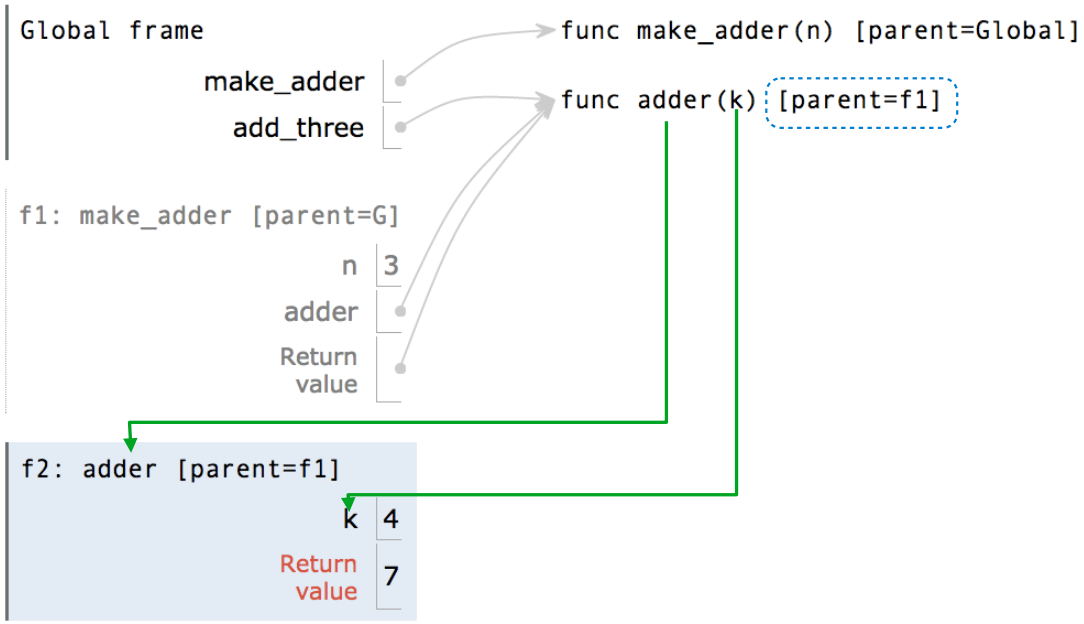

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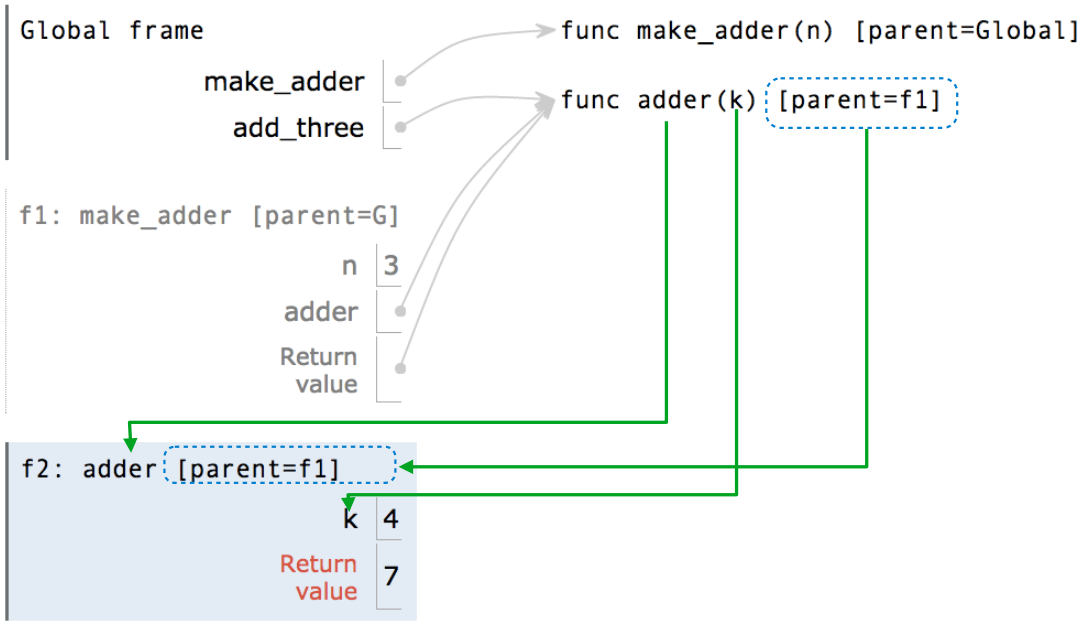

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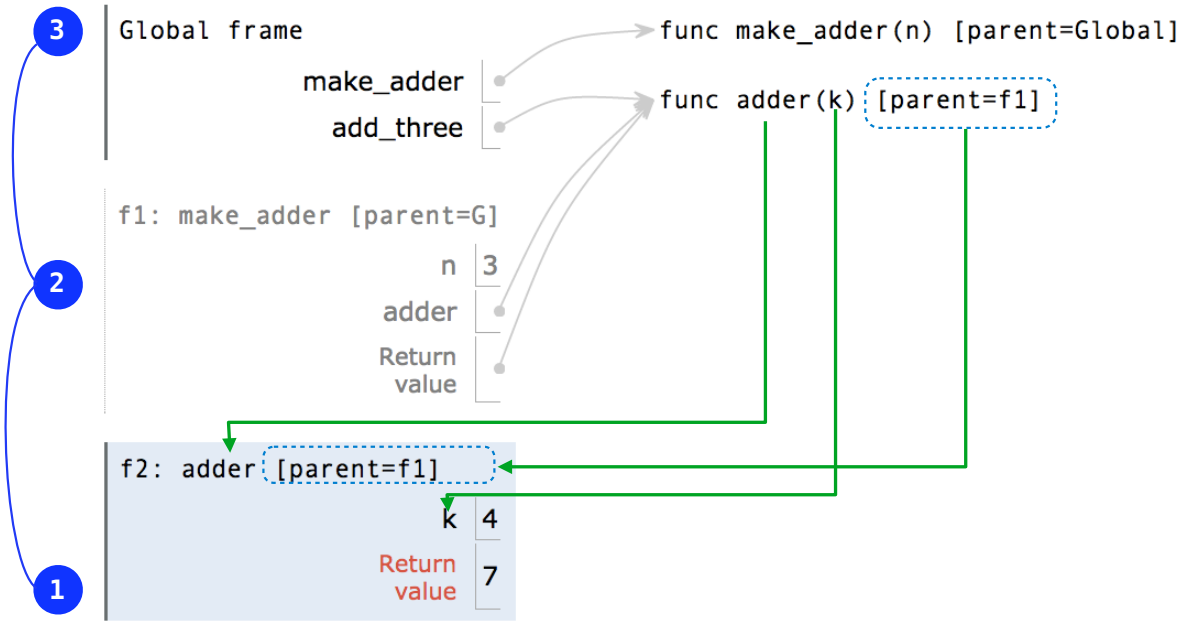

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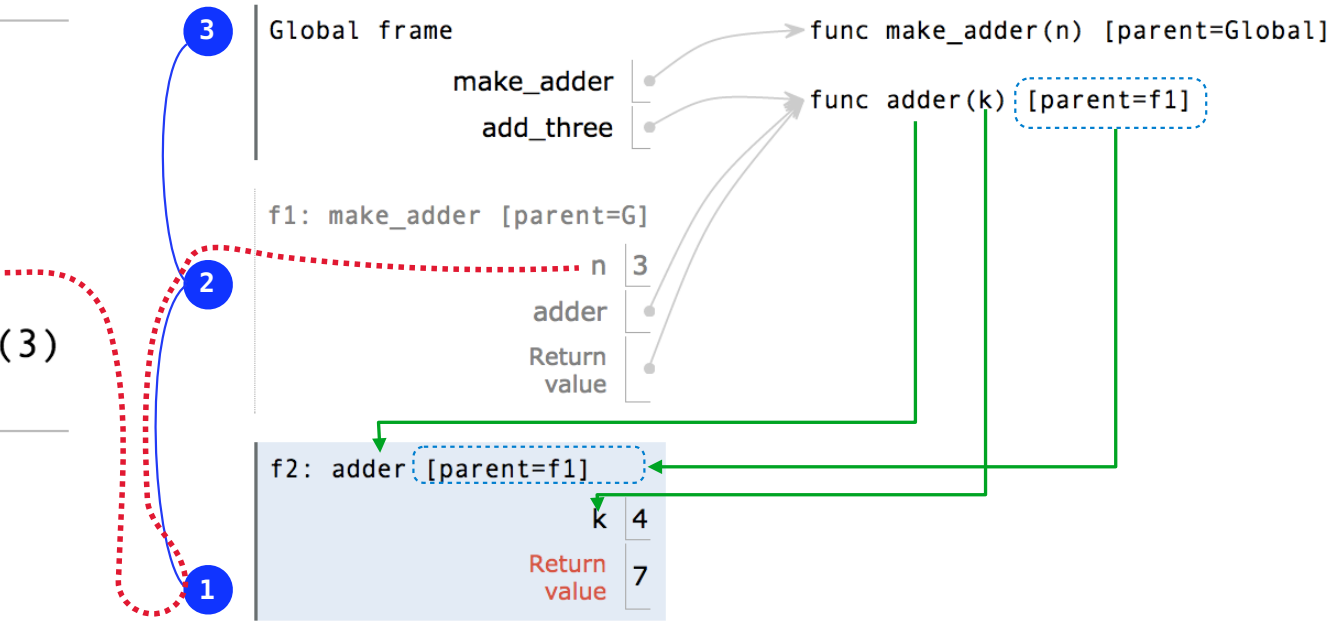

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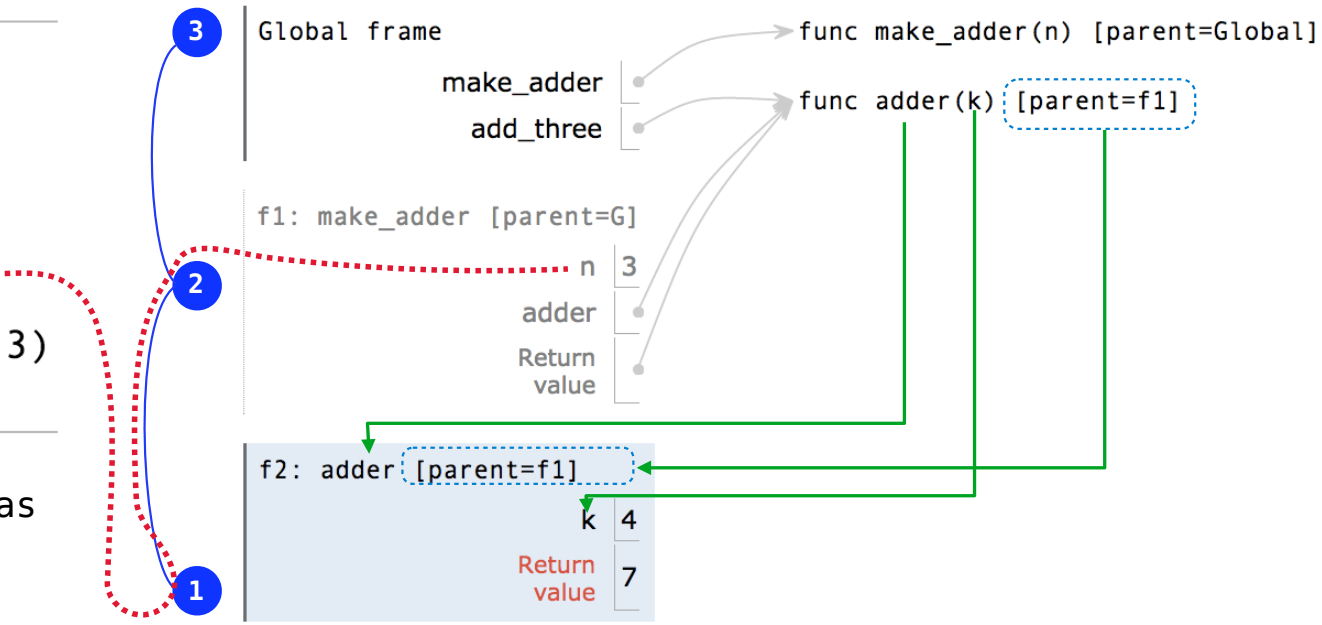

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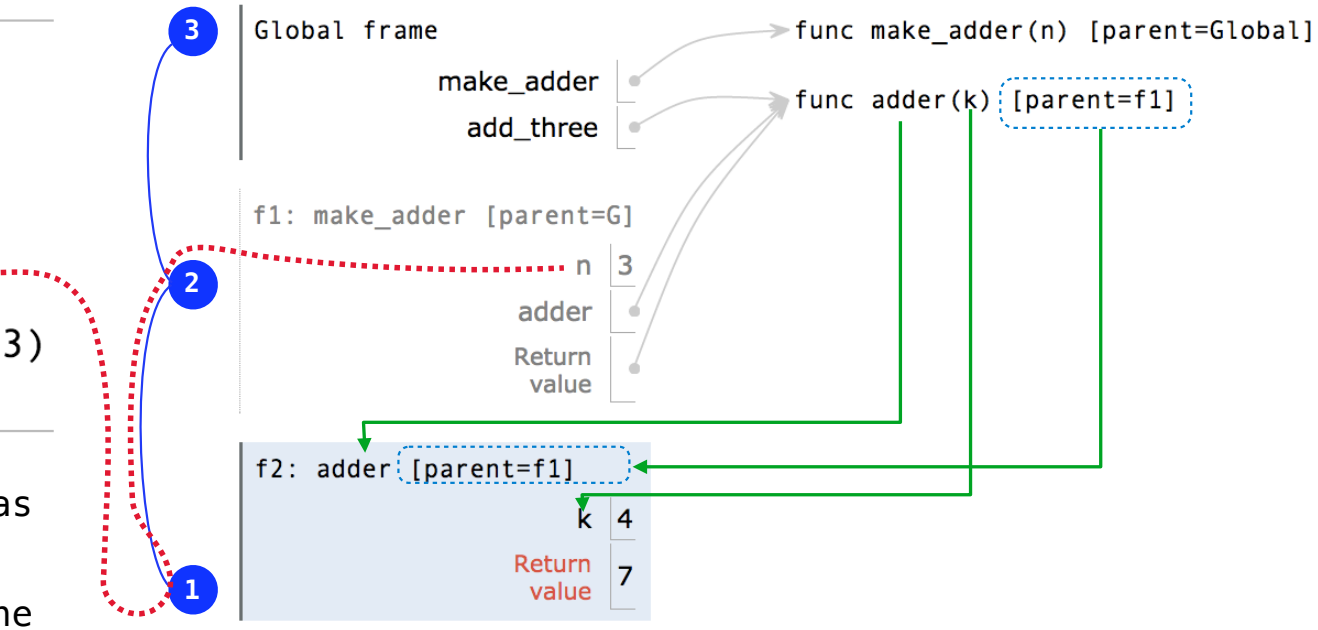

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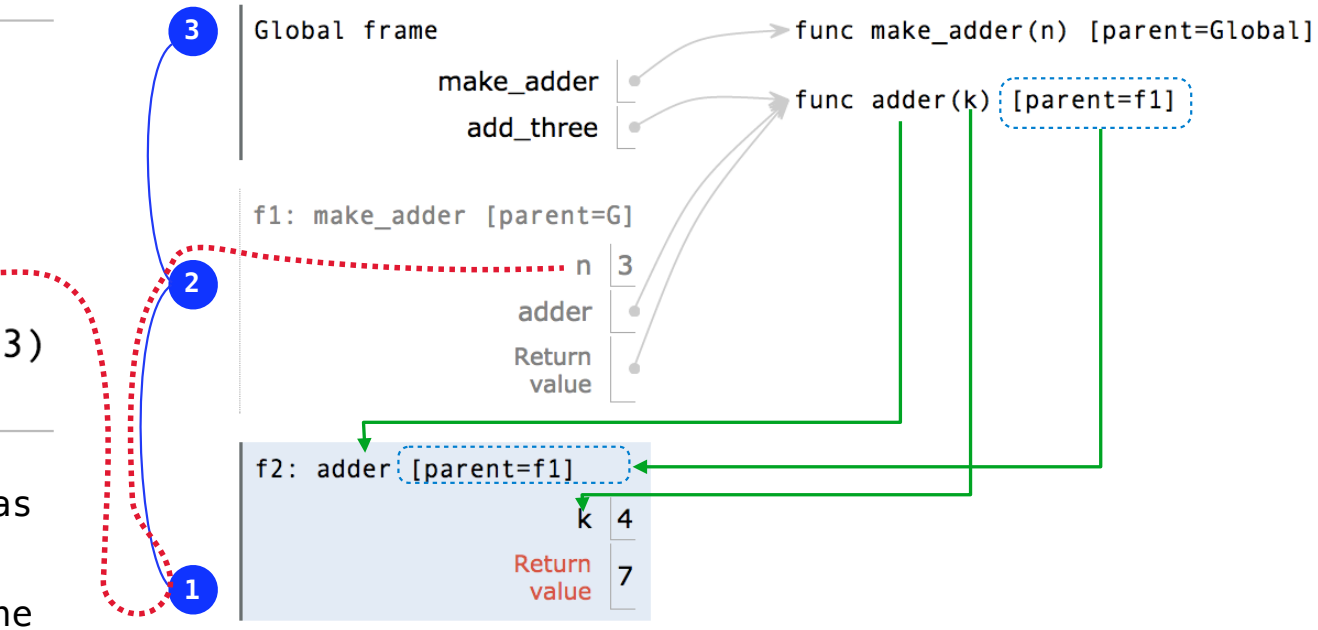

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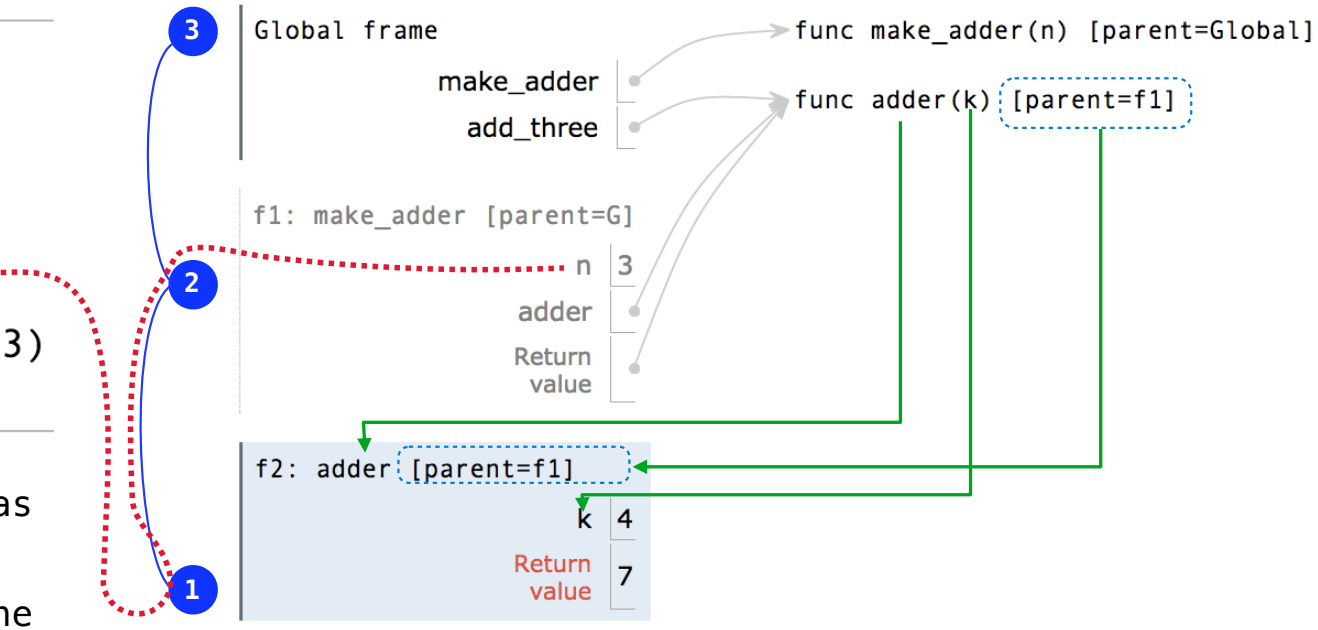

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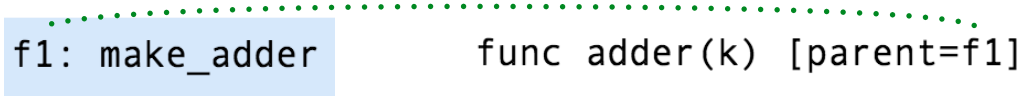

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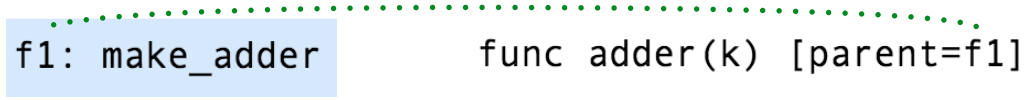

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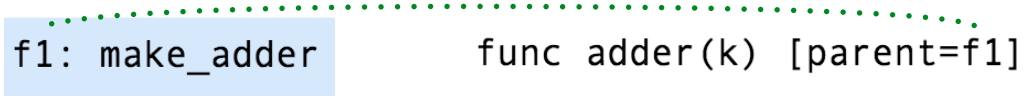

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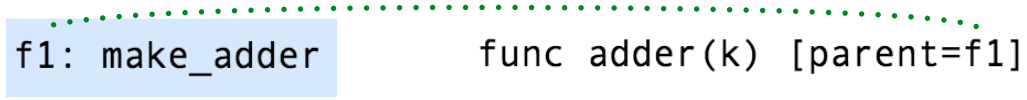

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1. Add a local frame, titled with the `<name>` of the function being called.

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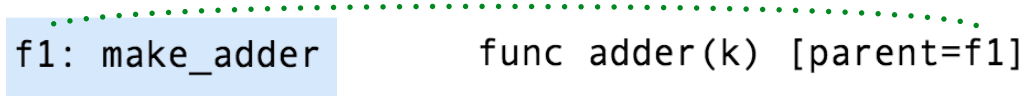

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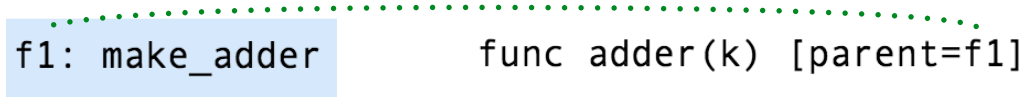
1. Add a local frame, titled with the `<name>` of the function being called.
2. Copy the parent of the function to the local frame: `[parent=<label>]`

How to Draw an Environment Diagram

When a function is defined:

Create a function value: func <name>(<formal parameters>) [parent=<label>]

Its parent is the current frame.



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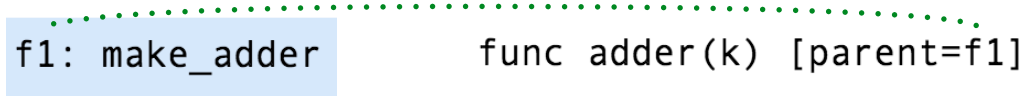

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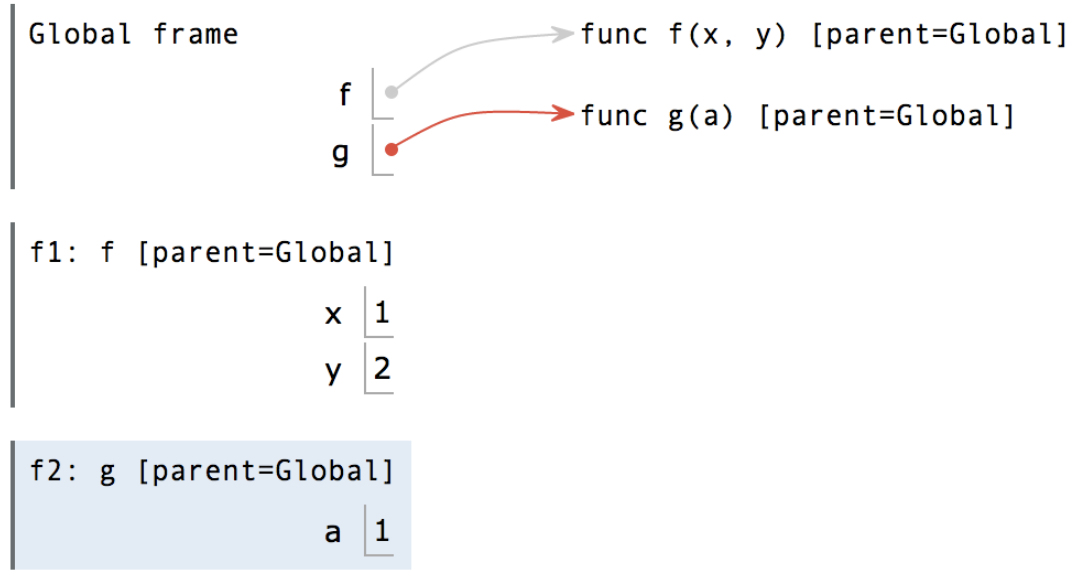
4. Execute the body of the function in the environment that starts with the local frame.

Local Names

(Demo)

Local Names are not Visible to Other (Non-Nested) Functions

```
1  def f(x, y):
2      return g(x)
3
4  def g(a):
5      return a + y
6
7  result = f(1, 2)
```



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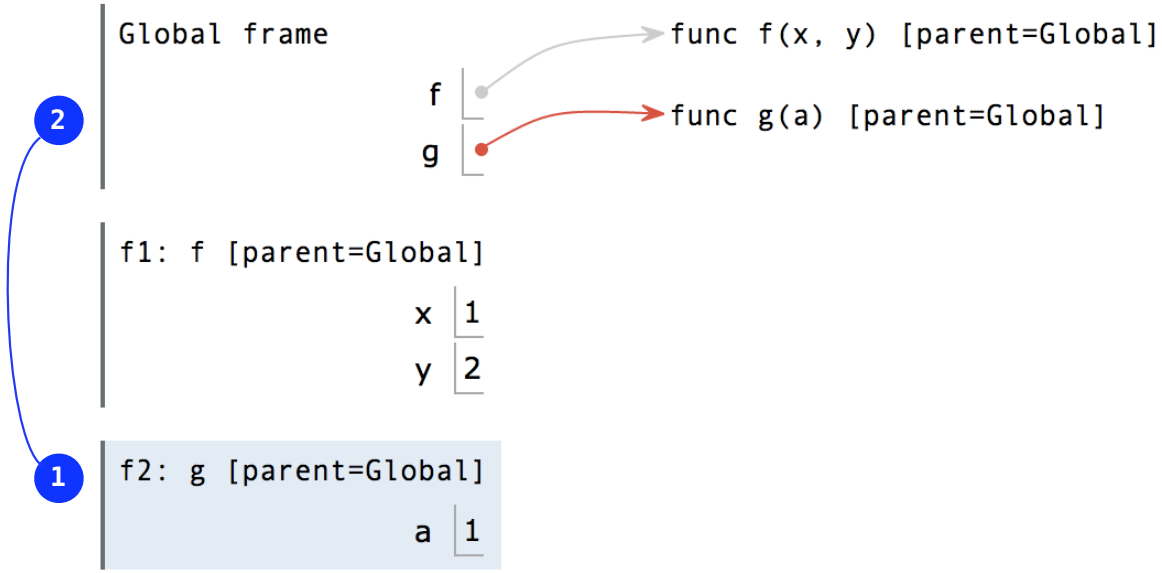

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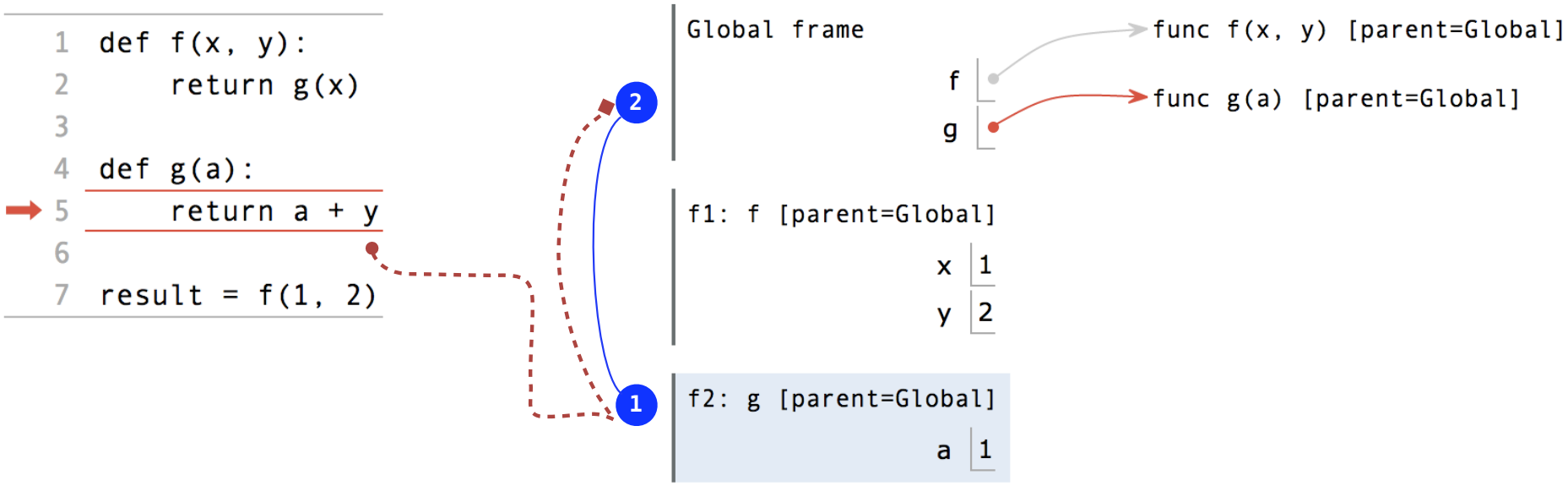

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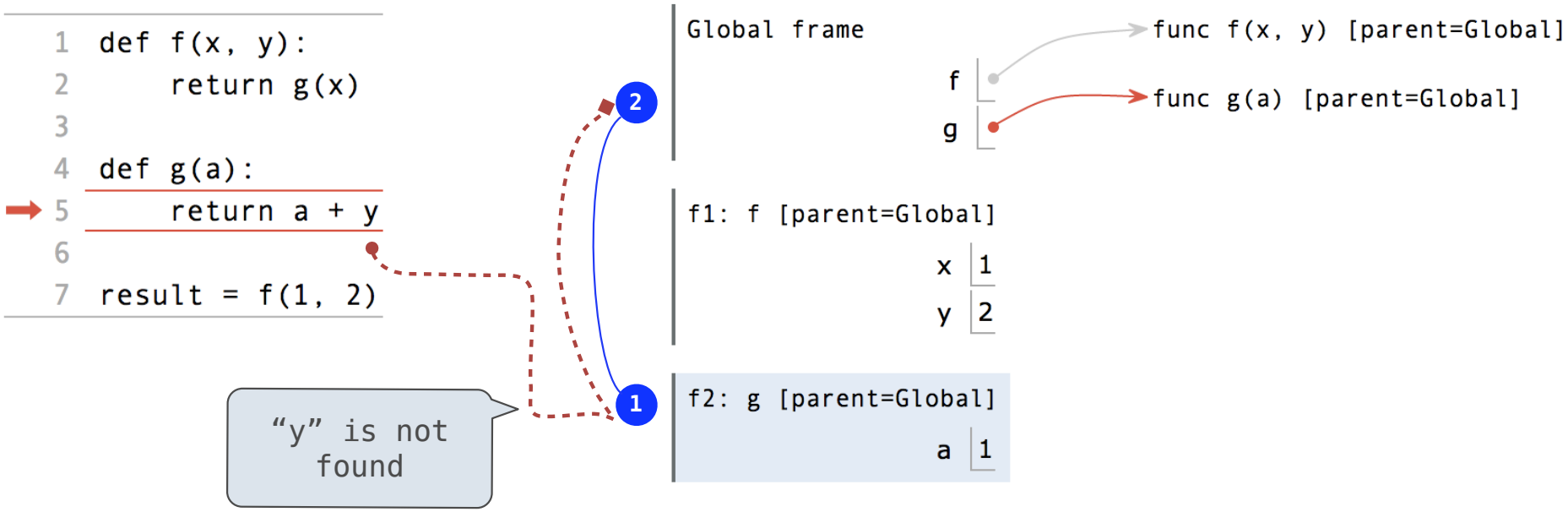

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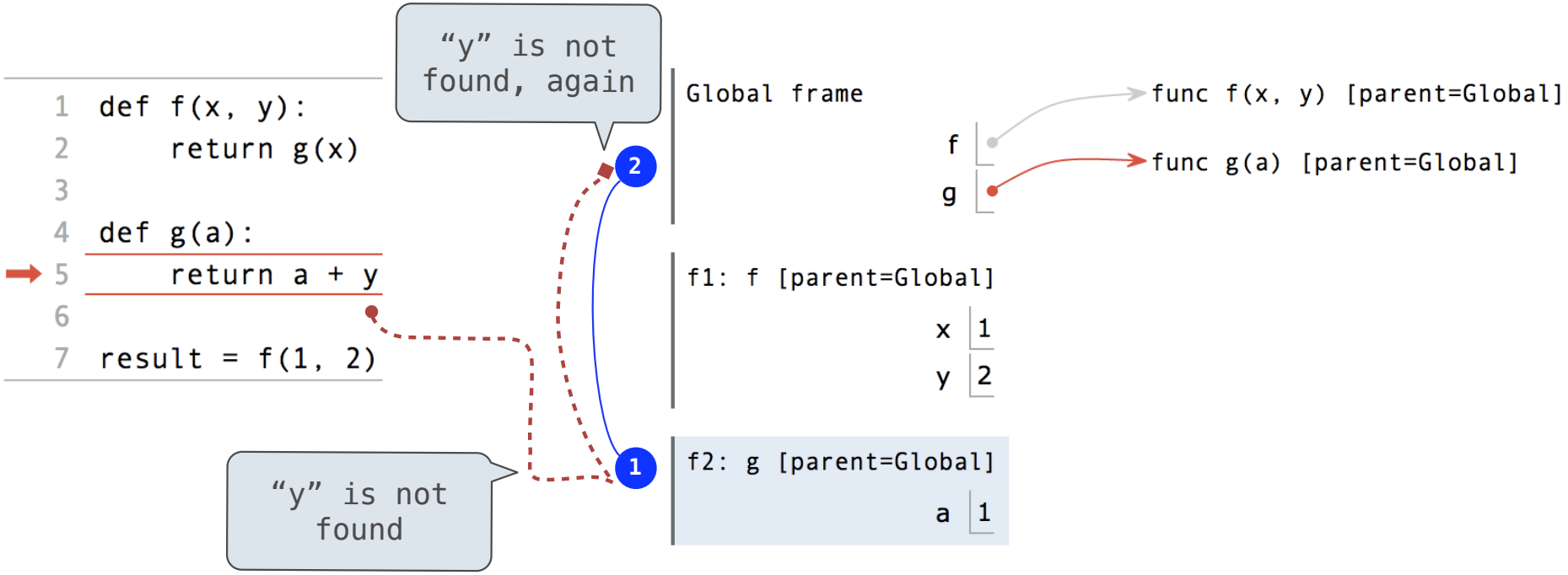

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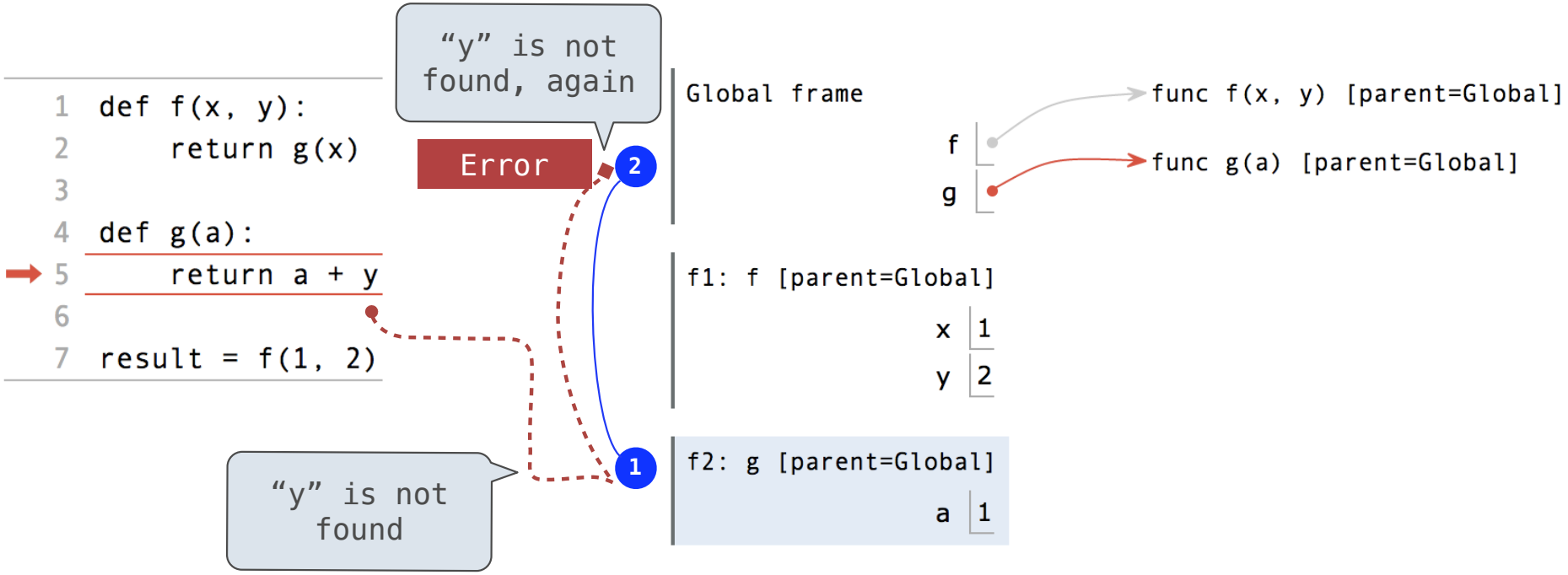

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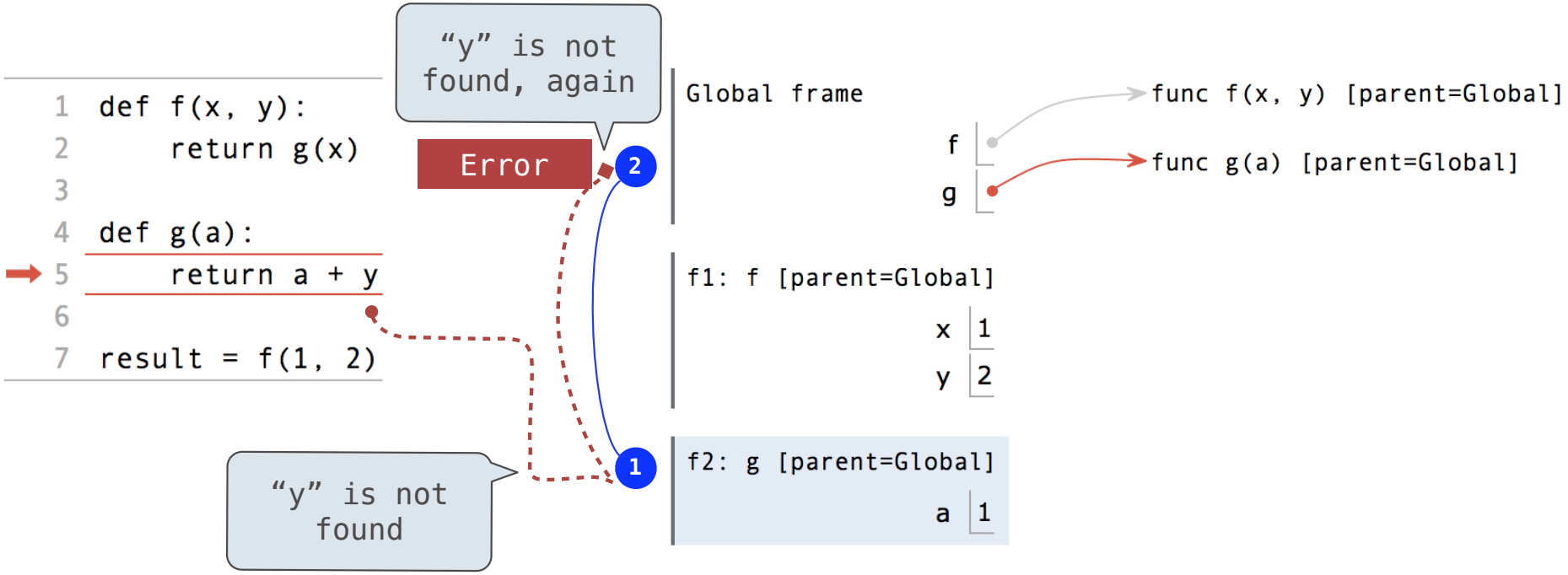

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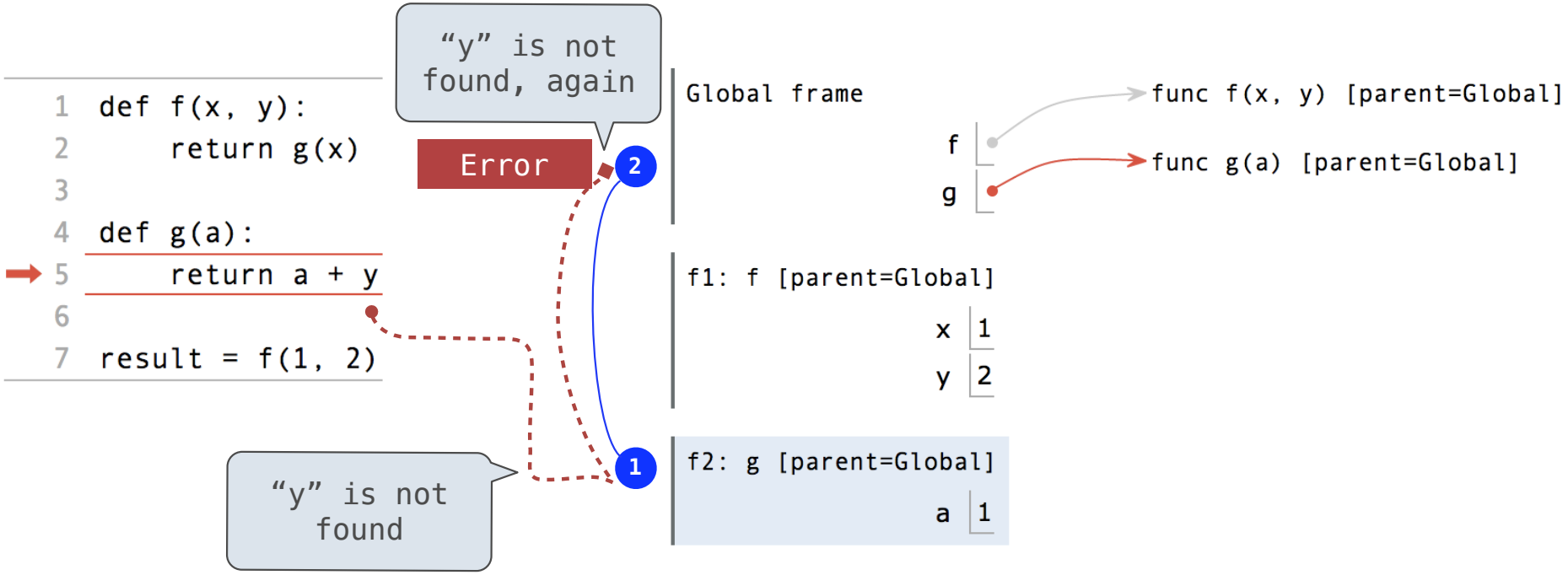

- An environment is a sequence of frames.
- The environment created by calling a top-level function (no def within def) consists of one local frame, followed by the global frame.

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Lambda Expressions

(Demo)

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Lambda expressions in Python cannot contain statements at all!

Lambda Expressions Versus Def Statements

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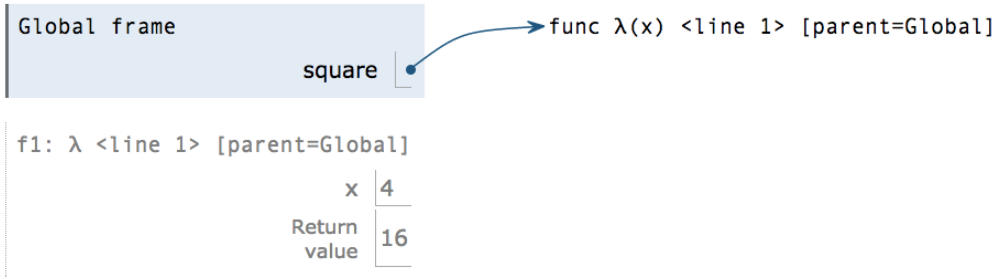

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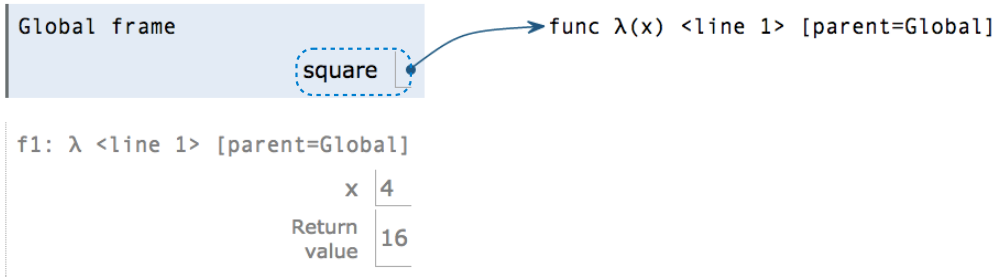

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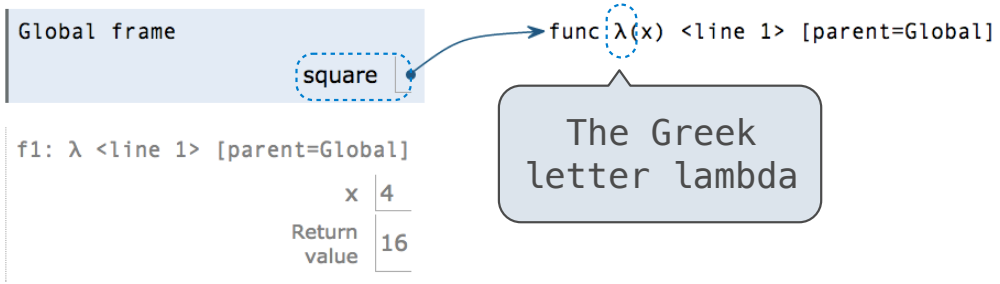

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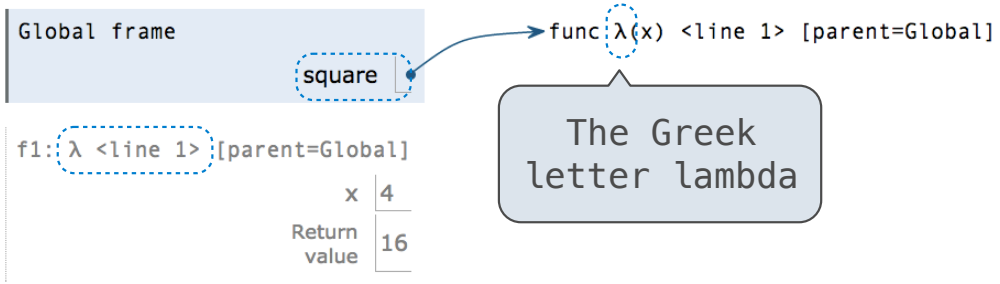

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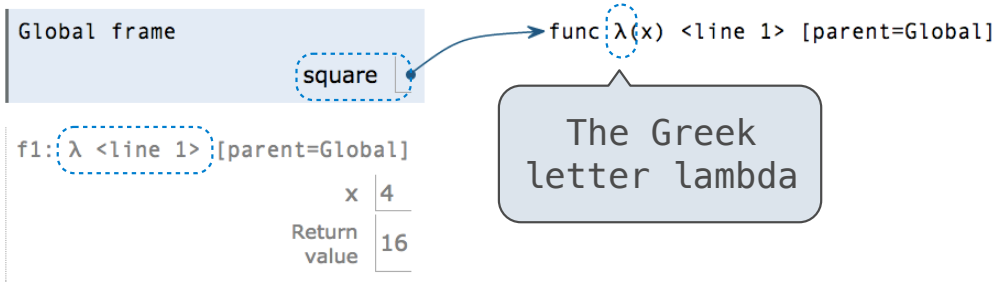

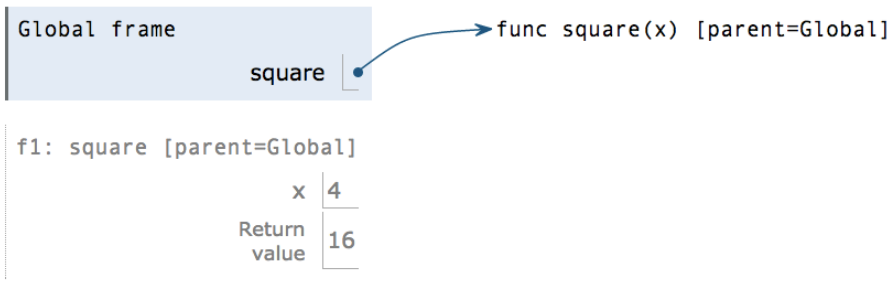

Function Composition

(Demo)

The Environment Diagram for Function Composition

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1  def square(x):
2      return x * x
3
4  def make_adder(n):
5      def adder(k):
6          return k + n
7      return adder
8
9  def compose1(f, g):
10     def h(x):
11         return f(g(x))
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14 compose1(square, make_adder(2))(3)
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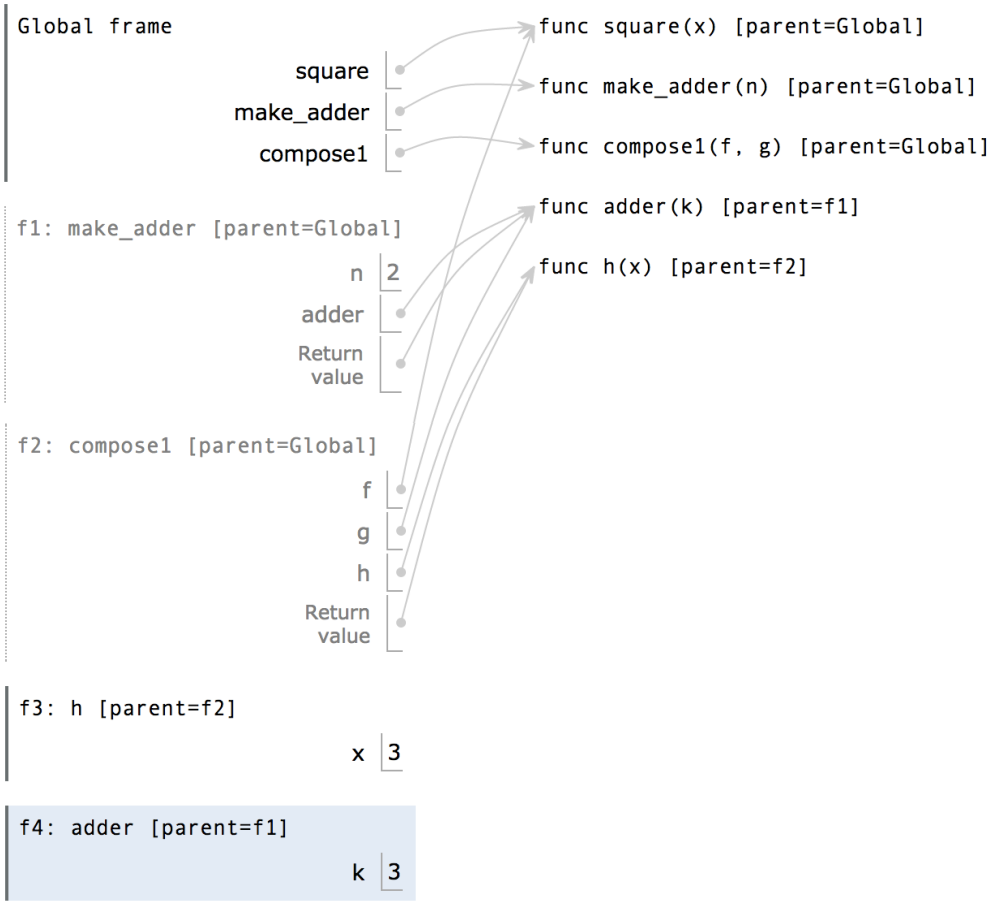

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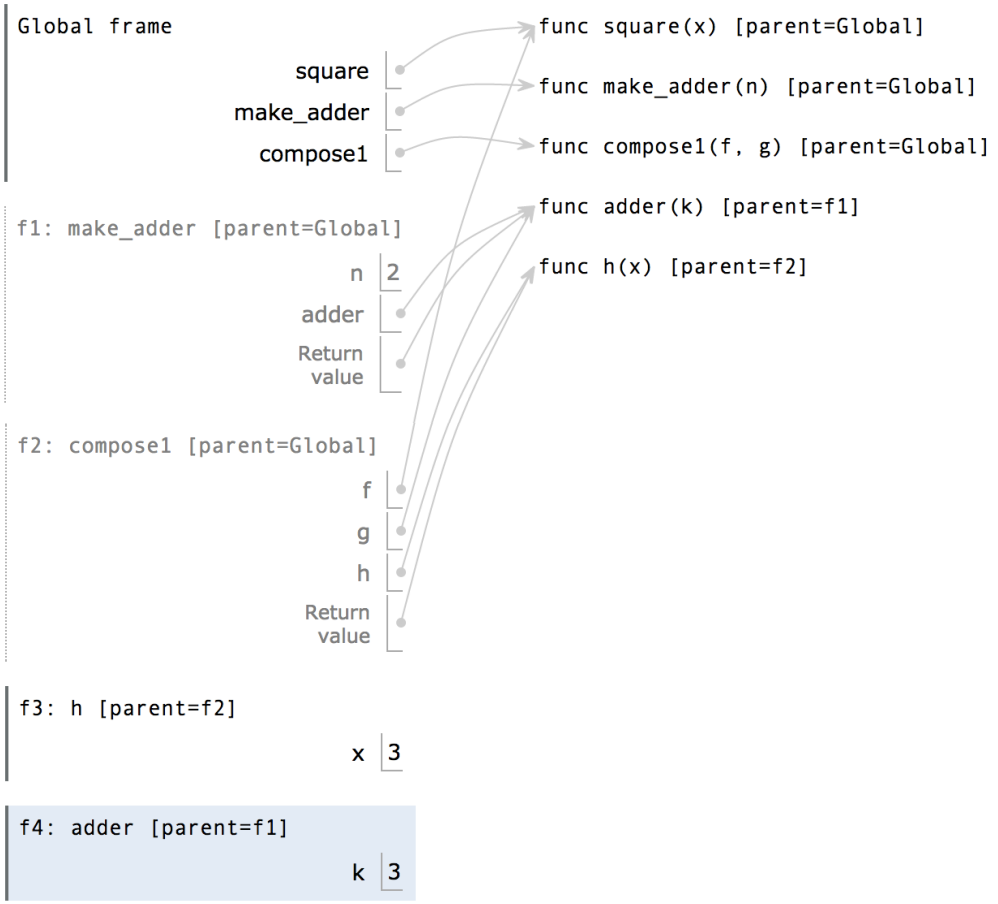

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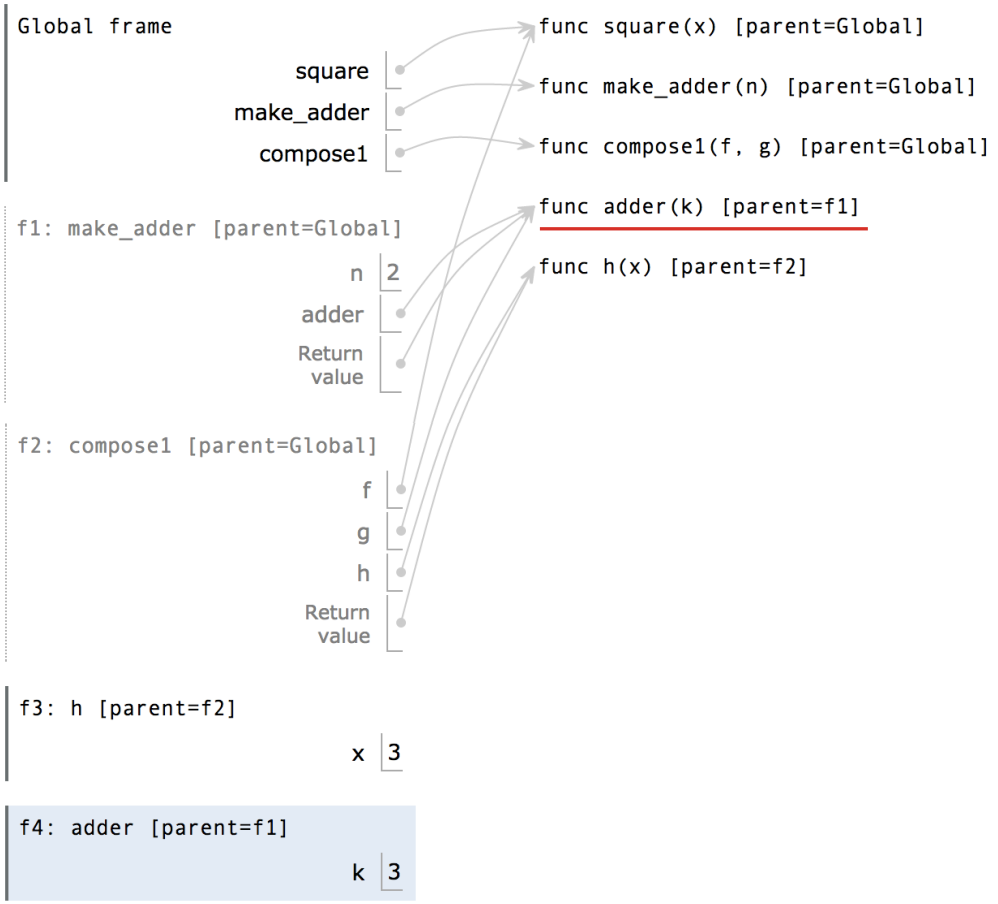

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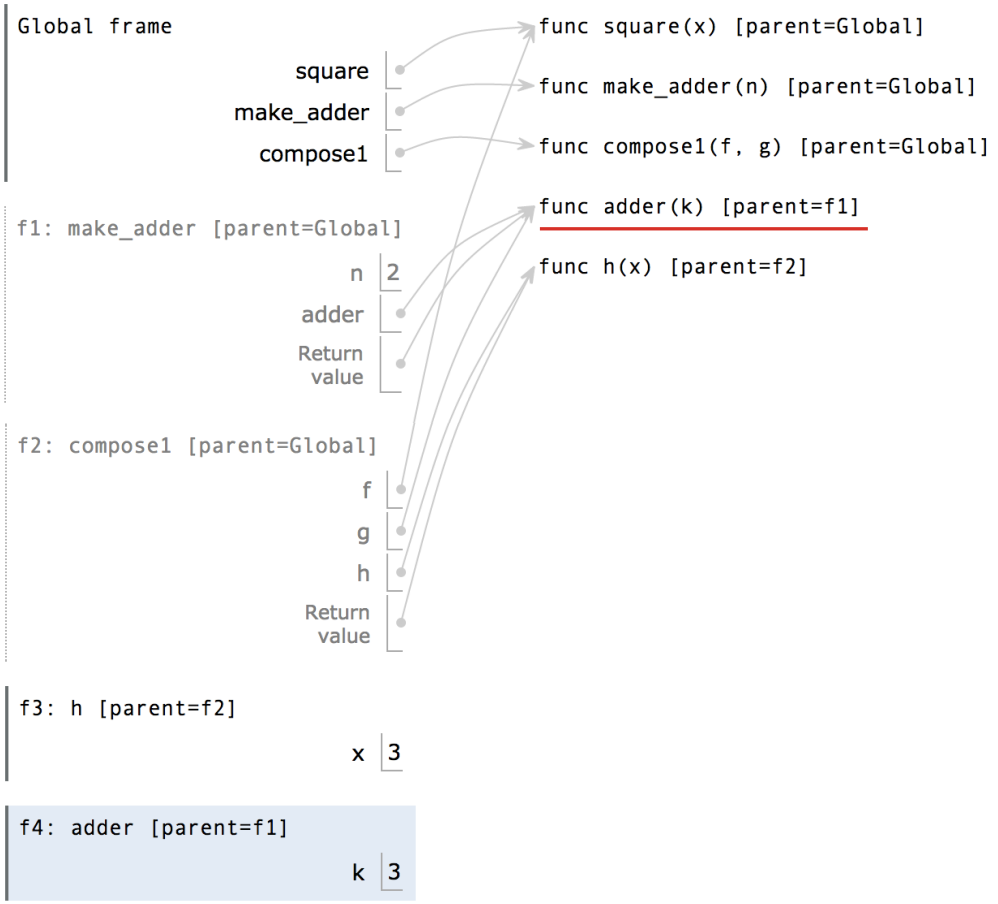

http://pythontutor.com/composingprograms.html#code=def%20square%28x%29%3A%0A%20%20%20%20return%20x%20*%20x%0A%0Adef%20make_adder%28n%29%3A%0A%20%20%20%20def%20adder%28k%29%3A%0A%20%20%20%20%20%20%20%20return%20k%20%2B%20n%0A%20%20%20%20return%20adder%0A%20%20%20%20%0Adef%20compose1%28f,%20g%29%3A%0A%20%20%20%20def%20h%28x%29%3A%0A%20%20%20%20%20%20%20%20return%20f%28g%28x%29%29%0A%20%20%20%20return%20h%0A%20%20%20%20%0Acompose1%28square,%20make_adder%282%29%29%283%29&cumulative=true&curInstr=0&mode=display&origin=composingprograms.js&py=3&rawInputLstJSON=%5B%5D

The Environment Diagram for Function Composition

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1  def square(x):
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14 compose1(square, make_adder(2))(3)
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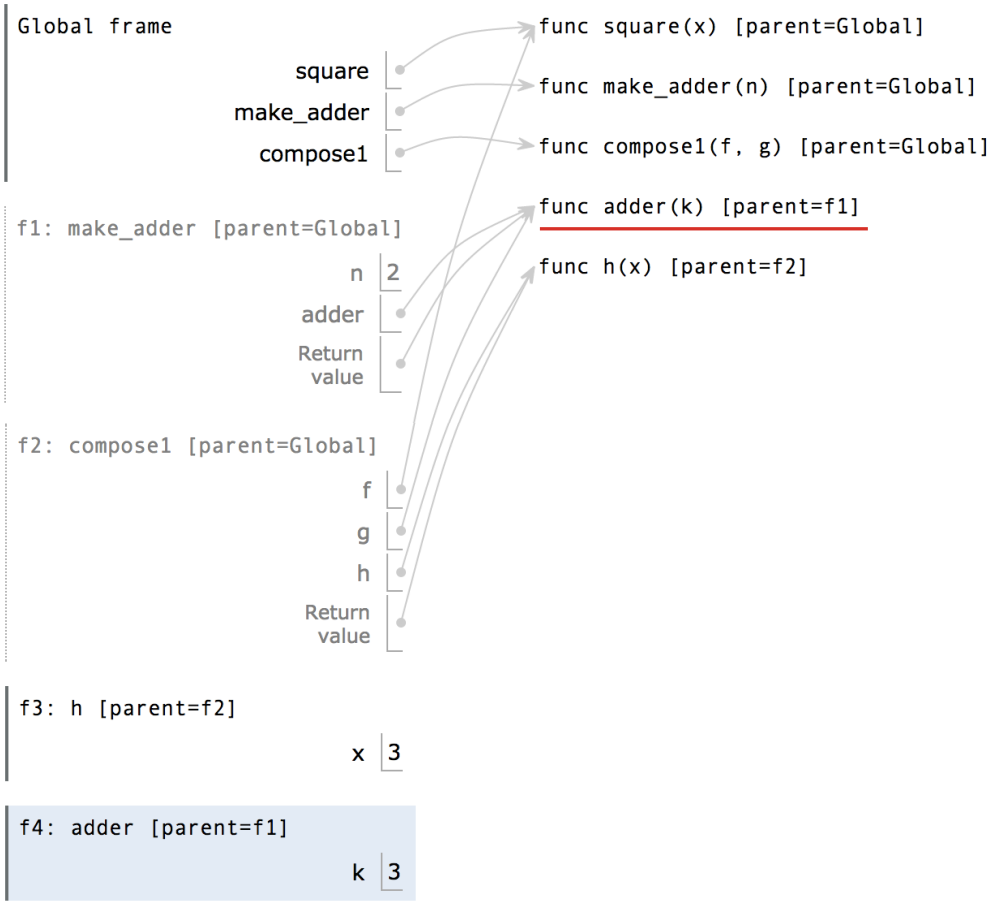
Return value of make_adder is an argument to compose1



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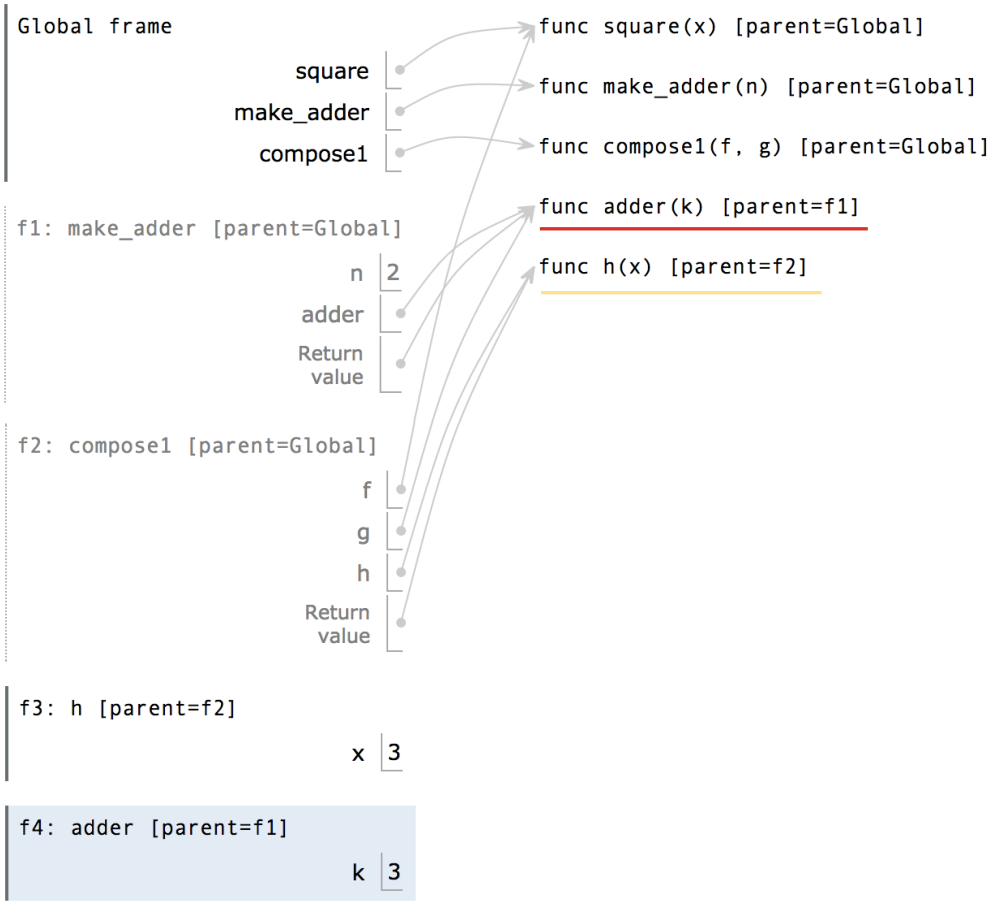

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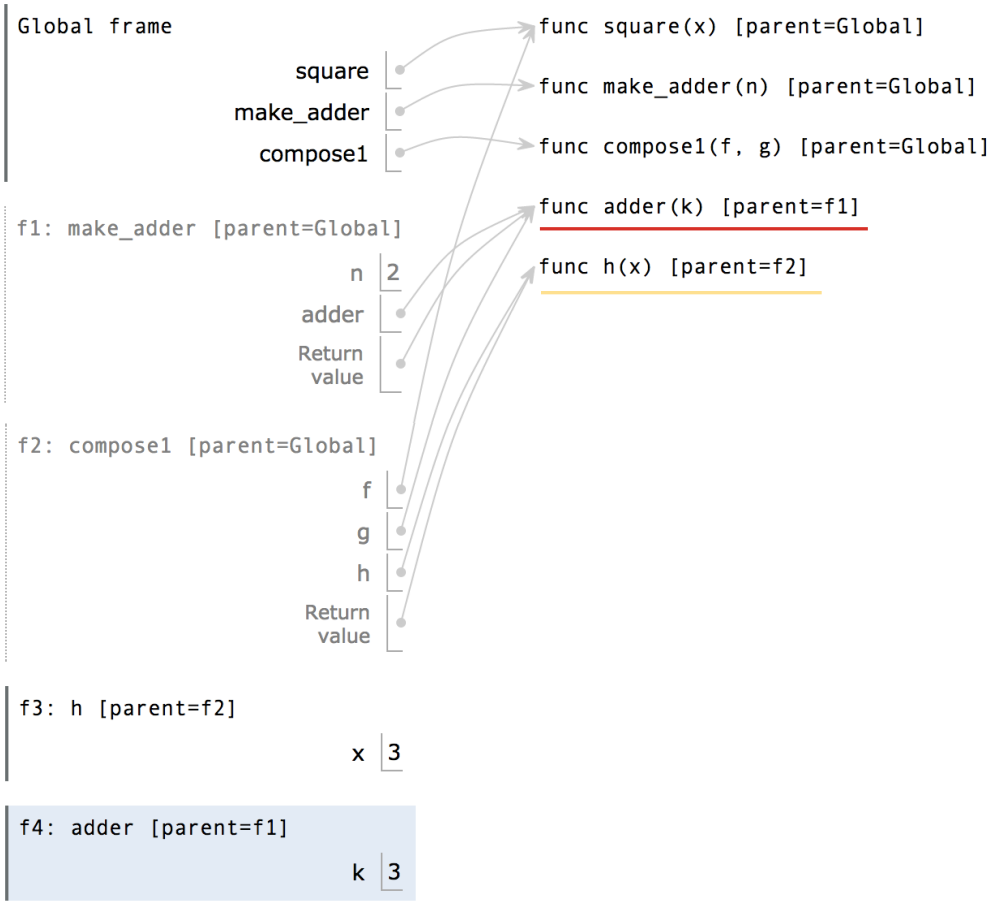

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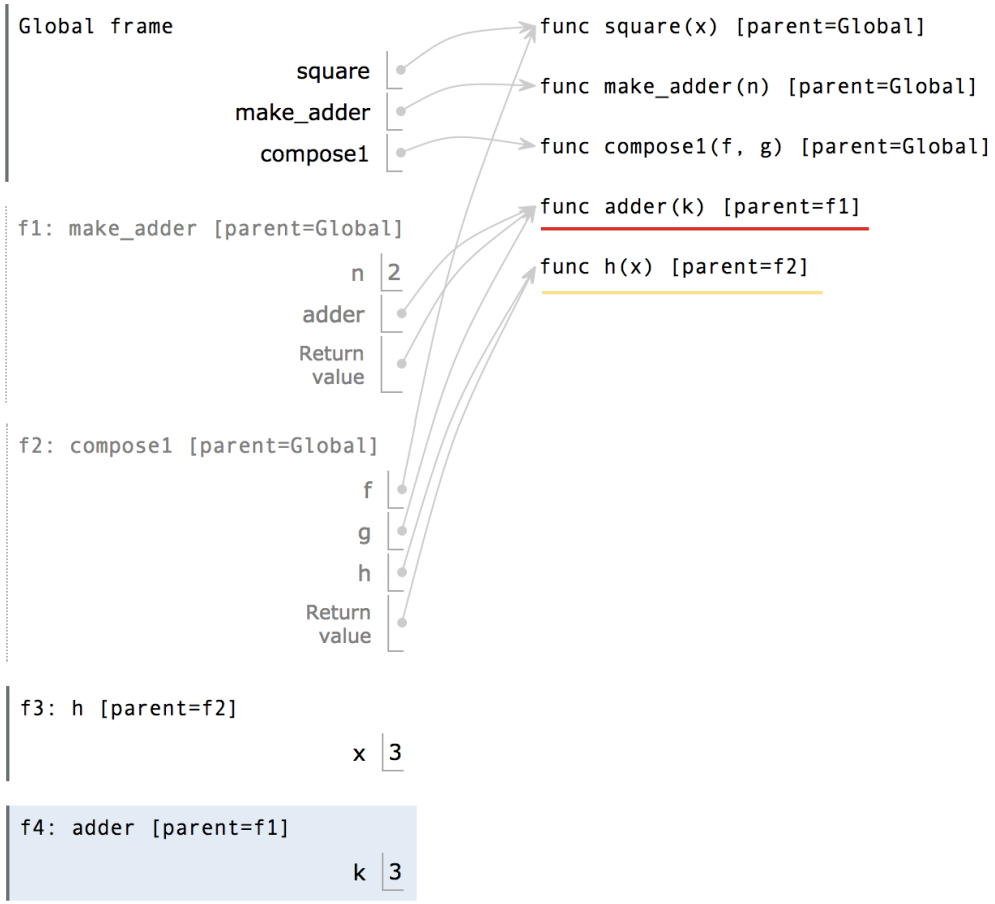

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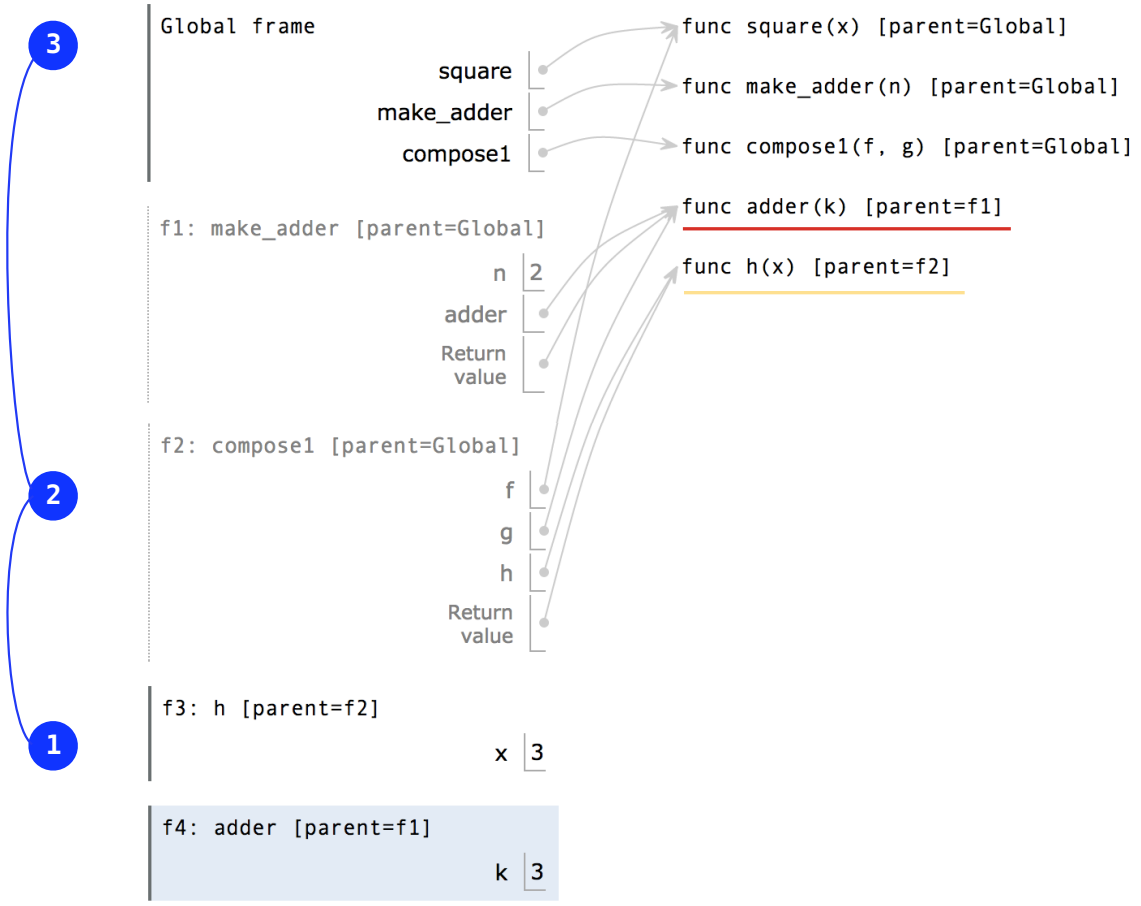

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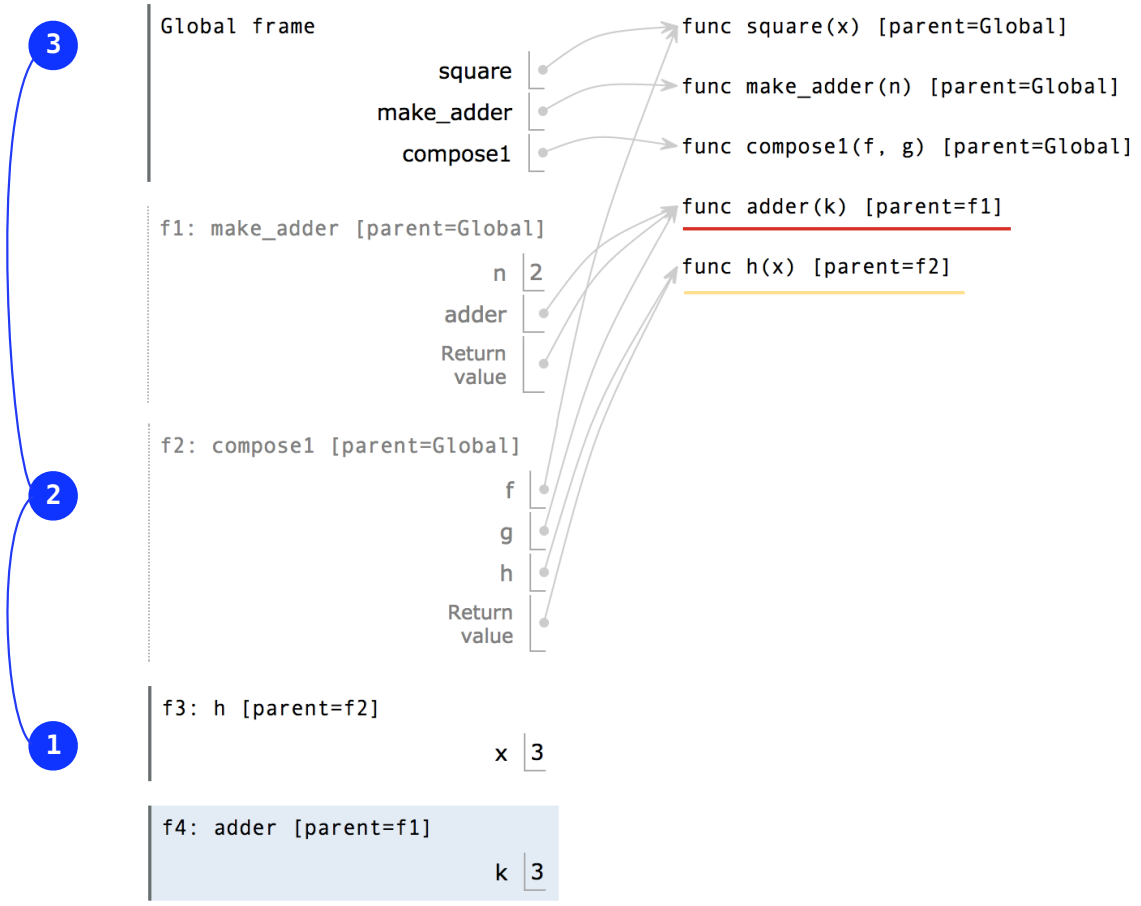

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